## Mathematical Modeling of Frictional Force between Ligature and Orthodontic Wire

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Received 24 November 2003; received in revised form 1 February 2005; accepted 7 February 2005

## **Abstract**

The main objective of this study is to present a mathematical model for frictional forces in orthodontic tooth movements. In order to produce lighter and more efficient sliding movement, good estimation of frictional forces must be determined. For the purpose of analysis, a typical bracket with a circular cross section arch wire is considered with a view to examine the effect of ligation technique (shape), geometry and properties of ligature, and arch wire on the friction forces between ligature and arch wire. Both uniform and non-uniform distributions of contact forces are considered. The result presented herein indicated that, for circular orthodontic arch wires, friction force between ligature and wire is proportional to tensile force in elastomeric ligature. This force is depended on the shape of ligation, material properties and geometries of wire, bracket and ligature.

Keywords: Ligature; Friction force; Mathematical models; Circular orthodontic wire; Tooth movement

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<sup>1</sup>Anchorage <sup>2</sup>Ligation <sup>3</sup>Self ligation

$$\operatorname{Ln}\frac{T_1}{T_2} = \mu\theta \quad \rightarrow \quad T_1 = T_2 e^{\mu \theta} \tag{)}$$

 $\mu = \mu_{lw}$ 

 $T_1$  $T_2$ 

[]  $W(\theta)$  ,

$$W(\theta) = \frac{T_2}{r} e^{\mu \theta} \tag{)}$$

θ

 $N = \int_{S_1}^{S_2} W(\theta) ds = \int_{\theta_1}^{\theta_2} T_2 e^{\mu \theta} d\theta$ 

 $\theta_2 = -\theta_I = \theta$ 

,( )

 $N_{n} = \frac{T_{2}}{\mu_{lw}} (e^{\mu_{lw}\theta} - e^{-\mu_{lw}\theta})$ ( ) ()

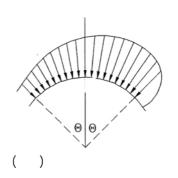
()  $N_u = 2T_2\theta$ 

()  $f_{lw} = N_i \mu_{lw} \quad , \quad i = n, u$ 

 $N_{i}$ 

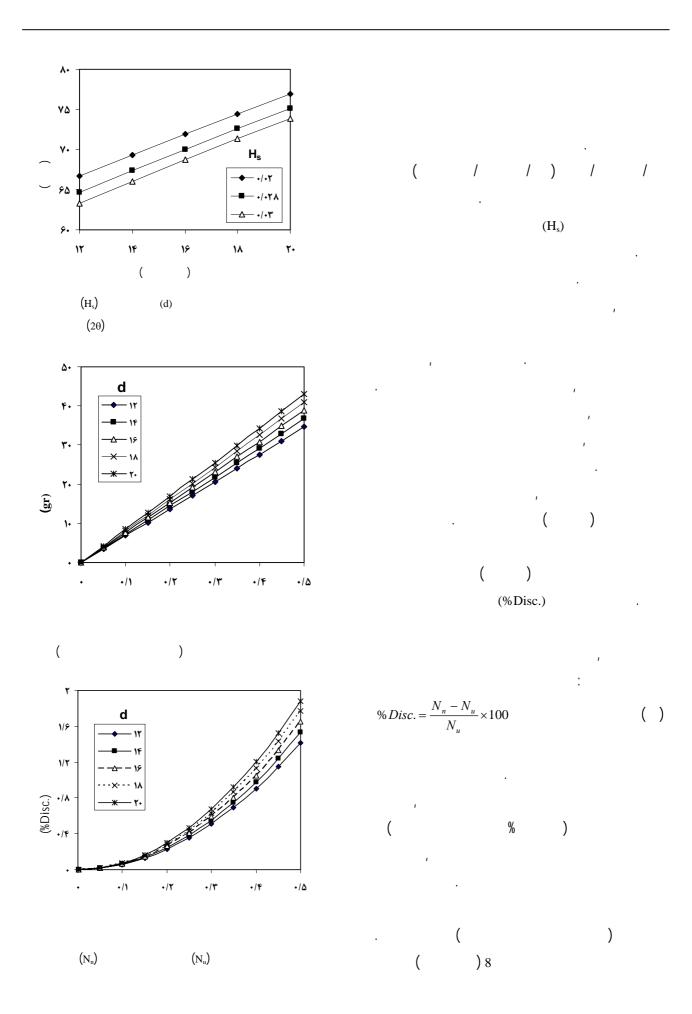
T<sub>2</sub> (

 $T_2$  $f_{\rm lw}$ 3



) (  $\sigma=E\times\epsilon$  $\varepsilon = \frac{1}{3}(\lambda - \lambda^{-2})$ ( )  $\sigma = \frac{E}{3}(\lambda - \lambda^{-2})$ ( ) Е  $T_{2}$ σ ( )  $T_2 = \sigma A_0$  $S_s$  $H_{\rm s}$  $A_0$  $f_{lw}$   $T_2$  $f_{\rm lw}$  . ()  $\Delta L = L_f - L_0 = 2(2BC - W_b) - 2\pi\,R$  $f_{lw} = \frac{1}{3} E A_0 (\lambda - \lambda^{-2}) (e^{\mu_{lw}\theta} - e^{-\mu_{lw}\theta})$  $L_f \quad L_0$ ,R  $f_{lw} = \frac{2}{3} E A_0 \,\mu_{lw} (\lambda - \lambda^{-2})$ ( )  $W_b$ . ABC BC $BC = \sqrt{AC^2 + AB^2}$ ()  $BC = \sqrt{(0.5S_b + 0.5d)^2 + (2r + d - 0.5H_s)^2}$ 3 λ  $\varepsilon = \frac{\Delta L}{L_0} = \lambda - 1$ () .[ ]  $d_f = d.\lambda^{-0.5}$  $0<\mu_{lw}<0.5$ ( )  $L_f = \lambda . L_0$ ( )  $d_f$  d  $T_{2} \\$ σ ) (

<sup>&</sup>lt;sup>4</sup>Rubberlike Elasticity



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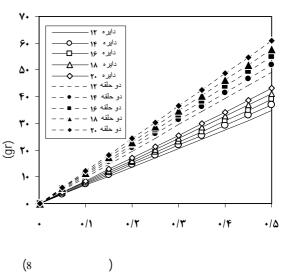
 $8 \qquad . \qquad . \\ L_f = 4(BC + \sqrt{BC^2 + 0.25W_b^2}) \qquad \qquad ( \\ 8 \qquad \qquad (\% \qquad )$ 

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